Mathematical Modelling of COVID-19 in Nigeria and its Numerical Solution using Variational Iteration Method (VIM)

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Abstract: In this paper, the modified SEIR Mathematical model for the outbreak and control of COVID-19 in Nigeria is presented. The model shows the difference between pre-symptomatic and asymptomatic Infectious diseases. This study used the SEIR model infection transmission dynamics incorporating asymptomatically and symptomatically infectious individuals, the final dynamics such as birth rate and mortality rate, facemask use diagnosis of asymptomatic infectious individuals and isolation of infected individuals as control strategy are also incorporated. The governing system of non-linear differential equations is solved using the variational iteration method (VIM) as an efficient analytical technique for approximating solutions to complex epidemiological models. We also determined the feasible region for the positive solution of the pandemic, the equilibrium points in the feasible region and to derive the general reproductive number for the system. Variational Iteration Method (VIM) is employed to give the approximate solution of the proposed problem. The study highlights the potential of VIM as an alternative approach for solving non-linear epidemic models and offers valuable predictions for policymakers in Nigeria. The results provide insights into the disease progression, control strategies and the impact of interventions such as quarantine, hospitalization and vaccination.

Keywords: SARS-CoV-2, COVID-19, pre-symptomatic Infectious, Variational Iteration Method, SEIR model.

Introduction

The Coronavirus pandemic, otherwise called the Covid epidemic, is an on-going worldwide wellbeing emergency brought about by the novel SARS-CoV-2 infection which leads to sickness known as coronavirus disease 2019. The epidemic was first found in Wuhan, China in December 2019. On January 30, 2020, the World Health Organization declared the scourge a Public Health Emergency of International Concern, and on March 11, it formally classified it as a pandemic. Over 9.15 million COVID-19 cases have been recorded across more than 188 countries and territories, leading to more than 473,000 deaths, as of June 23, 2020,. Additionally, more than 4.58 million people have be cured. (Wikipedia, n.d).

Nigeria, the most heavily populated Black nation with a population exceeding 200 million, reportedly affirmed the first occurrence of COVID-19 on February 27, 2020. The victim, an Italian national who work in Nigeria, had gotten back to Lagos from Milan, Italy, on February 25, 2020. At present, there is no specific cure for COVID-19; however, many of its symptoms can be cured. Treatment is tailored to the patient's clinical condition, with supportive care proving to be highly effective for those affected by the virus (Samuel, 2022). Since then, the number of cases has increased in Nigeria. As at 21st June, 2020, Nigeria has 20,919 COVID-19 CONFIRMED cases, 7109 cases have been cleared and 525 deaths have been documented in 35 states and the Federal Capital Territory. As of June 21, 2020, a total of 117,569 tests have been conducted (Ncdc, n.d). (Refer to Figure 1: Cumulative number of cases in Nigeria as of June 21, 2020, Figure 2: Cumulative Number of Recovered and Deaths in 8 States of Nigeria as at 21st of June, 2020 and Figure 3: Sources of exposure to COVID-19 in Nigeria as at 21st June, 2020.

In a struggle to battle the transmission of the epidemic disease, The Federal government of Nigeria conduct the cessation of all mobility in Lagos, Ogun and the FCT for an first phase of 14 days, which became effective from 11pm on Monday, 30th March, 2020 and on April 13, prolonged it by another 2 weeks. The government therefore temporarily suspended the lockdown with some businesses resuming operations from May 4, 2020, the government therefore imposed a nationwide curfew from 8pm to 6pm across the country as a feature of the new measures to limit the transmission of COVID-19. This is accompanied with the gradual relaxation of lockdown measures in FCT, Lagos and Ogun States (Samuel, 2022) and mandatory putting on of face masks in public and observing of body temperatures. However, restrictions, involving an overnight restriction and an outlaw on unnecessary highway travel, remain in check (Enahoro *et al*, 2020).

In this research, we originated a transformed SEIR model to truly understand the situation of things in Nigeria, to differentiate between the pre-symptomatically infectious and symptomatically infectious individual, to control the epidemic and to solve the equations using variational iteration scheme.

The variational iteration scheme initiated by Ji Huan, is a powerful and reliable method for both numerical and analytical purposes. This scheme has been universally used by mathematicians to simplify linear and nonlinear, as we as homogeneous and inhomogeneous scientific and engineering problems such as linear and nonlinear Klein Gordon equation (Usman *et al.*, 2019), uniform and non-uniform partial differential equations (Olayiwola, 2015) the relativistic Klein-Gordon equation (Usman *et al.*, 2019), SIR model (Akinboro, *et al.*, 2014), (Gabariela and Sudi, 2019), Typhoid Fever model (Adebisi, *et al.*, 2018), infectious disease model (Ayoade *et al.*, (2019) and many more. It has been shown that the solution scheme is effective and reliable for all types of differential equations.

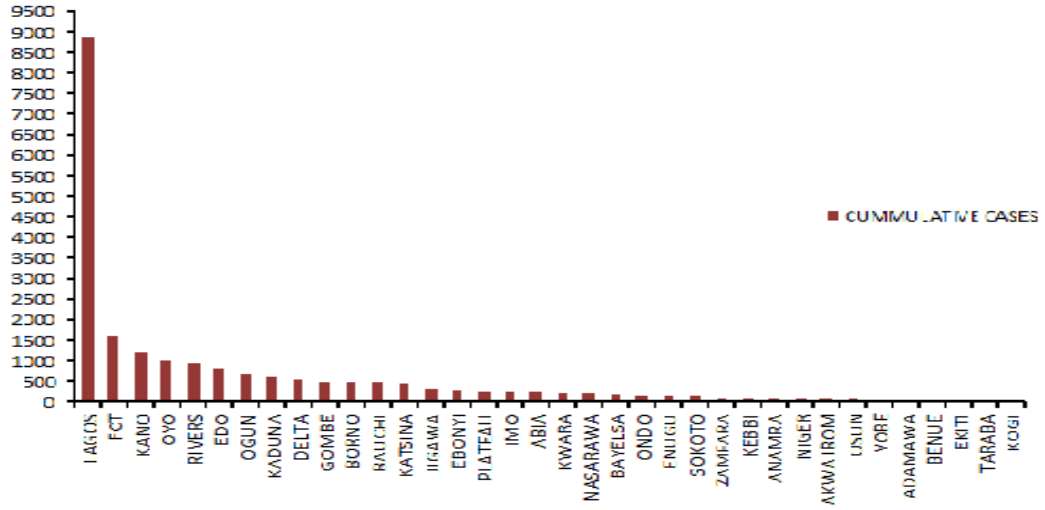


Figure 1: Cumulative number of cases in Nigeria as at June 21, 2020
(Data source: covid-19.ncdc.gov.ng/)

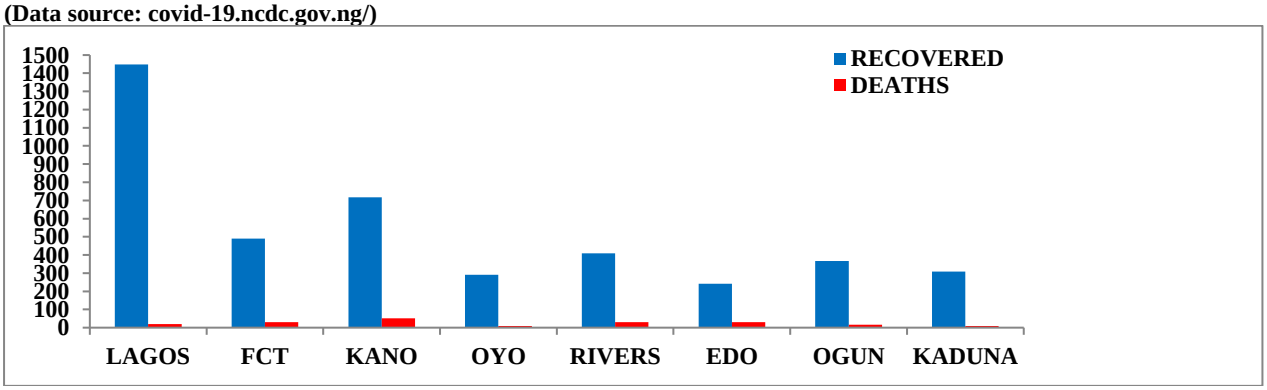


Figure 2: Cumulative Number of Recovered and Deaths In 8 States of Nigeria As At June 21, 2020. DATA SOURCE: covid-19.ncdc.gov.ng/

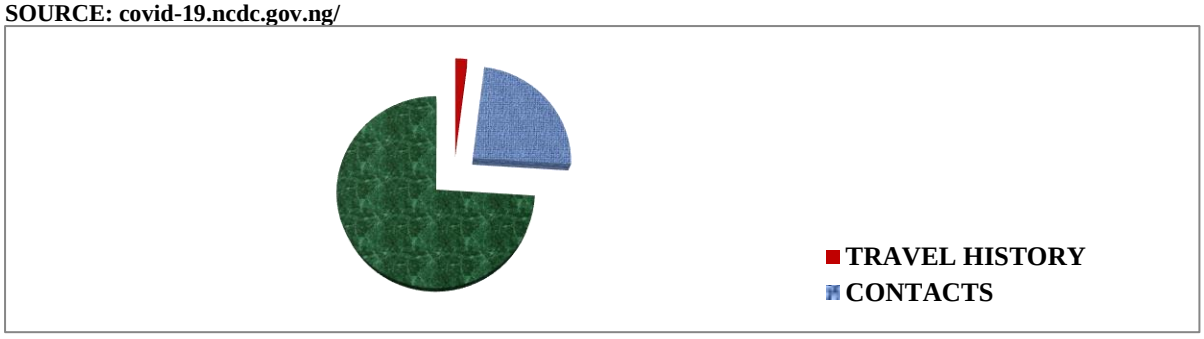


Figure 3: Sources of exposure to COVID-19 in Nigeria as at 21st June, 2020
(Data Source: Nigeria Centre for Disease Control (NCDC)

Mathematical Modeling

Mathematical modelling involves using mathematical concepts and terminology to represent a system. A model may help in explaining what the system is all about, to study the effect of different classes in the model and to make predictions.

Many researchers at present are modelling COVID-19 coronavirus. For example (Jiajun *et al.*, 2020) suggested using the SEIR model that the governments must enforce the isolation system rigorously to successfully control the dissemination of the disease during the pandemic, (Enahoro *et al.*, 2020) created a mathematical representation to analyse the dissemination of disease and management of COVID-19 in Nigeria, (Oianying *et al*, 2020) proposed a theoretical models for the COVID-19 epidemic in Wuhan, taking into account individual behavioural actions and government actions, (Peilang and Kang, 2020) presented a modified SEIR model with time lag influence and distribution of probability of model states.(Roman and Vasyl, 2020) proposed a model to described the quantitative of the outbreak of COVID-19, (Jose *et al.*, 2020) implemented the SEIR model to calculate the number of individual infected and the number of casualties of COVID-19, (Leonard and Xavier, 2020) used the SEIR model to forecast the outbreak ofCOVID-19 in Spain, (Widijaningsih *et al*, 2018) used the SEIR model with immigration to determine the equilibrium points, (Ayub *et al*, 2020) solved the COVID-19 model equation using three powerful numerical

methods: Euler's method, second and fourth order Range Kutta methods, (Chinwendu *et al*, 2020) applied the model to control the dissemination of COVID-19 and many more.

Model Formulation

This section outlines the problem design. The formulation is established by dividing the entire population at a time (t), represented by N(t), into seven population segments namely: susceptible S(t), exposed E(t), Pre-Symptomatic Infectious (I_p), Symptomatic Infectious (I_s), Asymptomatic Infectious (I_a), Hospitalized (H) and Cured individuals (R). Individuals who have never experienced COVID-19 are added into the "vulnerable" populace through birth or immigration at pace of β and susceptible individuals are "introduced" to the epidemic by encountering an infected individual at the rate γ and reduced by natural death at a rate μ, after a set of incubation period, they become "Infectious". Some individuals have shorter latency periods and are infectious before they display symptoms, they are "pre-symptomatic", some individuals never display any symptoms, i.e. they are "asymptomatic". We also have another group, another set of individuals who are highly contagious, they spread the virus like wildfire, the "symptomatic", the pre-symptomatic, after about 1-3 days of incubation, also move to the 'symptomatic" at the rate α_4. The symptomatic individuals including pre-symptomatic individuals will be moved to the "Hospital" at the rate α_5 for treatment. Finally, individual from the asymptomatic moves to recovered compartment at the rate α_6, while an individual is "recovered" at the rate, α_7 after the treatment or after the infection has run its course but diminished by natural death at a rate μ and also reduced by death from the disease at a rate δ.

Therefore,

$$N(t) = S(t) + E(t) + I_p(t) + I_s(t) + I_a(t) + H(t) + R(t)$$

Assumptions of the Model

1. Individual in each sub-population may experience a inherent death at the rate, μ,
2. A recovered individual cannot be re-infected with COVID-19.
3. α_1, α_2, α_3 be the transmission rates for pre-symptomatic, symptomatic and asymptomatic respectively.
4. The individuals died of the virus at the rate, δ.

Differences between Asymptomatic and Pre-symptomatic Spread of COVID-19

Asymptomatic Transmission refers to the dissemination of the plague by individual who are yet to exhibit symptoms and will never develop any sign of infection. However, these individual can still infect others making them unwell.

Pre-symptomatic Transmission is the spread of the virus by individual who are yet to exhibit symptoms, but will eventually develop them later. These individual can unknowingly infect others before showing the sign of sickness.

Pre-symptomatic spread occurs more frequently than asymptomatic transmission and this is possible due to the fact that individual can transmit the virus through common behaviour such as speaking, which sometimes can involve micro-droplet touching their face, and then coming in contact with surfaces. This will allow the virus to be transmitted to others without having the symptoms. (Holly, 2020)

Pictorial Representation of the Model

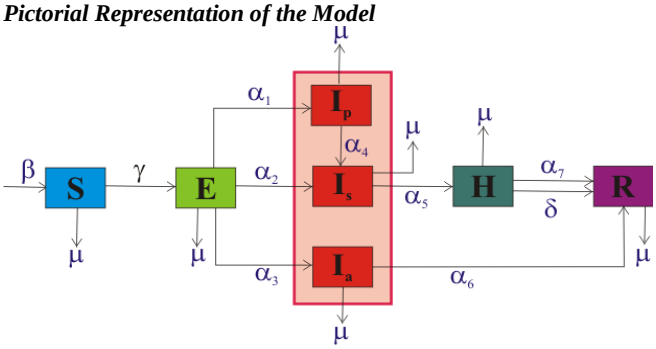


Figure 4: Pictorial Representation of the Model

Equations of the Model

$$\frac{dS}{dt} = S' = \beta - (\gamma S I_P + \gamma S I_S + \gamma S I_a) - \mu S \quad (1)$$

$$\frac{dE}{dt} = E' = \gamma S I_P + \gamma S I_S + \gamma S I_a - (\alpha_1 E + \alpha_2 E + \alpha_3 E) - \mu E \quad (2)$$

$$\frac{dI_p}{dt} = I_p' = \alpha_1 E - I_p(\alpha_4 + \mu) \quad (3)$$

$$\frac{dI_s}{dt} = I_s' = \alpha_2 E + \alpha_4 I_p - I_s(\alpha_5 + \mu) \quad (4)$$

$$\frac{dI_a}{dt} = I_a' = \alpha_3 E - I_a(\alpha_6 + \mu) \quad (5)$$

$$\frac{dH}{dt} = H' = I_s\alpha_5 - \alpha_7 H - \delta H - \mu H \quad (6)$$

$$\frac{dR}{dt} = R' = \alpha_7 H + \delta H + \alpha_6 I_a - \mu R \quad (7)$$

Where $N(t) = S(t) + E(t) + I_a(t) + I_p(t) + I_s(t) + H(t) + R(t)$

Table 1: Description of the variables of the Model

Variable	Description
S(t)	**S**usceptible individuals at time *t*
E(t)	**E**xposed Individuals at time *t*
$I_p(t)$	**Pre**-symptomatically-Contagious individuals at time *t*
$I_s(t)$	**S**ymptomatically- Contagious individuals at time *t*
$I_a(t)$	**A**symptomatically-Contagious individuals at time *t*
H(t)	**H**ospitalized Individual at time *t*
R(t)	**R**emoved (Recovered+Dead) individuals at time *t*

Table 2: Description of the parameters of the Mode

Parameters	Description
β	Constant influx of new susceptible (through birth or immigration)
γ	Rate of Infection
μ	natural death rate (assumed the same)
α_1	Transmission rate between E and I_p
α_2	Transmission rate between E and I_s
α_3	Transmission rate between E and I_a
α_4	Transmission rate between I_p and I_s
α_5	Hospitalization rate between I_s and H
α_6	Recovery rate between I_a and H
α_7	Recovery rate between H and R
δ	COVID-19 induced death rate

Feasibility Region for A Positive Solution

Feasible Region $\sum$

$$\frac{dN}{dt} \geq 0$$

Consider the equation:

$$N(t)=S(t) + E(t) + I_p(t) + I_s(t) + I_a(t) + H(t) + R(t) \quad (8)$$

the increment of N(t), write *t* is given by:

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dI_p}{dt} + \frac{dI_s}{dt} + \frac{dI_a}{dt} + \frac{dH}{dt} + \frac{dR}{dt}$$

add equations (1-7)

$$\beta - (\gamma S I_P + \gamma S I_S + \gamma S I_a) - \mu S + \gamma S I_P + \gamma S I_S + \gamma S I_a - (\alpha_1 E + \alpha_2 E + \alpha_3 E) - \mu E + \alpha_1 E - I_p(\alpha_4 + \mu) + \alpha_2 E + \alpha_4 I_p - I_s(\alpha_5 + \mu) + \alpha_3 E - I_a(\alpha_6 + \mu) + I_s\alpha_5 - \alpha_7 H - \delta H - \mu H + \alpha_7 H + \delta H + \alpha_6 I_a - \mu R$$

$$\frac{dN}{dt} = \beta - \mu S + \mu E + \mu I_P + \mu I_S + \mu I_a + \mu H + \mu R$$

$$\frac{dN}{dt} = \beta - \mu(S + E + I_P + I_s + I_a + H + R)$$

from equation (8), $N = S + E + I_P + I_s + I_a + H + R$

$$\frac{dN}{dt} = \beta - N\mu \geq 0$$

$\beta - N\mu \geq 0$ if and only if $\beta \geq N\mu$

$\frac{\beta}{\mu} \geq N = (S + E + I_P + I_s + I_a + H + R)$

$$\sum = \left\{(S, E, I_P, I_s, I_a, H, R) \in /S(+E + I_P + I_s + I_a + H + R) \leq \frac{\beta}{\mu}\right\}$$

Equilibrium Points in the Feasible Region

We have two equilibrium points in the model: (1) Disease Free Equilibrium (2) Endemic Equilibrium. At equilibrium points, equations (1-7) must be equated to zero (0).

that is:

$$\frac{dS^*}{dt} = \frac{dE^*}{dt} = \frac{dI_p{}^*}{dt} = \frac{dI_s{}^*}{dt} = \frac{dI_a{}^*}{dt} = \frac{dH^*}{dt} = \frac{dR^*}{dt} = 0$$

From (3)

$$\alpha_1 E^* - I_p{}^*(\alpha_4 + \mu) = 0$$

$$I_p{}^* = \left(\frac{\alpha_1}{\alpha_4+\mu}\right) E^*$$

(9)

From (4)

$$\alpha_2 E^* + \alpha_4 I_p{}^* - I_s{}^*(\alpha_5 + \mu) = 0$$

$$I_s{}^* = \left(\frac{\alpha_2(\alpha_4+\mu)+\alpha_4\alpha_1}{(\alpha_4+\mu)(\alpha_5+\mu)}\right) E^*$$

(10)

From (5)

$$\alpha_3 E^* - I_a{}^*(\alpha_6 + \mu) = 0$$

$$I_a{}^* = \left(\frac{\alpha_3}{\alpha_6+\mu}\right) E^*$$

(11)

from (6)

$$I_s{}^*\alpha_5 - H^*(\alpha_7 + \delta + \mu) = 0$$

$$H^* = \alpha_5 \left(\frac{\alpha_2(\alpha_4+\mu)+\alpha_4\alpha_1}{(\alpha_4+\mu)(\alpha_5+\mu)(\alpha_7+\delta+\mu)}\right) E^*$$

(12)

from (7)

$$H^*(\alpha_7 + \delta) + \alpha_6 I_a{}^* - \mu R^* = 0$$

$$R^* = \left(\left(\frac{\alpha_5(\alpha_7+\delta)(\alpha_2+\alpha_4+\alpha_4\mu+\alpha_4\alpha_1}{(\alpha_4+\mu)(\alpha_5+\mu)(\alpha_7+\delta+\mu)}\right) + \left(\frac{\alpha_3\alpha_6}{\alpha_6+\mu}\right)\right)\frac{1}{\mu} E^*$$

(13)

CASE 1: Disease-Free Equilibrium

Equilibrium when $E^* = 0$, from (9-13)

$E^* = 0 \Rightarrow I_P{}^* = 0, I_S{}^* = 0, I_a{}^* = 0, H^* = 0, R^* = 0$

$\therefore$ from (1) $\Rightarrow$ $0 = \beta - (\gamma S^* I_P{}^* + \gamma S^* I_S{}^* + \gamma S^* I_a{}^*) - \mu S^*$

$$S^* = \frac{\beta}{\mu} \quad (14)$$

$Disease - free\ equilibrium\ P_0 = \left(\frac{\beta}{\mu}, 0, 0, 0, 0, 0, 0\right)$

CASE 2: Endemic Equilibrium

Equilibrium when $E^* \neq 0$, from (9-13)

$E^* \neq 0 \Rightarrow I_P{}^* \neq 0, I_S{}^* \neq 0, I_a{}^* \neq 0, H^* \neq 0, R^* \neq 0$

from (2)

$$0 = \gamma S^*(I_P{}^* + I_s{}^* + I_a{}^*) - E^*(\alpha_1 + \alpha_2 + \alpha_3 + \mu)$$
$$\gamma S^*(I_P{}^* + I_s{}^* + I_a{}^*) = E^*(\alpha_1 + \alpha_2 + \alpha_3 + \mu)$$

If $E^* \neq 0$ then

$$S^* = \frac{1}{\gamma}\left(\frac{(\alpha_1+\alpha_2+\alpha_3+\mu)(\alpha_4+\mu)(\alpha_5+\mu)(\alpha_6+\mu)}{\alpha_1(\alpha_5+\mu)(\alpha_6+\mu)+\alpha_3(\alpha_4+\mu)(\alpha_5+\mu)+\alpha_6+\mu)(\alpha_2\alpha_4+\alpha_2\mu+\alpha_4\alpha_1}\right) \quad (15)$$

Derive R_0, the General Reproductive Number

From (14), $S^* \geq \frac{\beta}{\mu}$

From (15), $S^* = \frac{1}{\gamma}\left(\frac{(\alpha_1+\alpha_2+\alpha_3+\mu)(\alpha_4+\mu)(\alpha_5+\mu)(\alpha_6+\mu)}{\alpha_1(\alpha_5+\mu)(\alpha_6+\mu)+\alpha_3(\alpha_4+\mu)(\alpha_5+\mu)+\alpha_6+\mu)(\alpha_2\alpha_4+\alpha_2\mu+\alpha_4\alpha_1}\right)$

$$\frac{1}{\gamma}\left(\frac{(\alpha_1+\alpha_2+\alpha_3+\mu)(\alpha_4+\mu)(\alpha_5+\mu)(\alpha_6+\mu)}{\alpha_1(\alpha_5+\mu)(\alpha_6+\mu)+\alpha_3(\alpha_4+\mu)(\alpha_5+\mu)+\alpha_6+\mu)(\alpha_2\alpha_4+\alpha_2\mu+\alpha_4\alpha_1}\right) \geq \frac{\beta}{\mu}$$

$$1 \geq \frac{\beta}{\mu\gamma}\left(\frac{(\alpha_1+\alpha_2+\alpha_3+\mu)(\alpha_4+\mu)(\alpha_5+\mu)(\alpha_6+\mu)}{\alpha_1(\alpha_5+\mu)(\alpha_6+\mu)+\alpha_3(\alpha_4+\mu)(\alpha_5+\mu)+\alpha_6+\mu)(\alpha_2\alpha_4+\alpha_2\mu+\alpha_4\alpha_1}\right) = R_0$$

$$R_0 = \frac{\beta}{\mu\gamma}\left(\frac{(\alpha_1+\alpha_2+\alpha_3+\mu)(\alpha_4+\mu)(\alpha_5+\mu)(\alpha_6+\mu)}{\alpha_1(\alpha_5+\mu)(\alpha_6+\mu)+\alpha_3(\alpha_4+\mu)(\alpha_5+\mu)+\alpha_6+\mu)(\alpha_2\alpha_4+\alpha_2\mu+\alpha_4\alpha_1}\right) \quad (16)$$

(i) $R_0 \leq 1 \Rightarrow$ **disease-free equilibrium**
$P_0(\frac{\beta}{\mu}, 0, 0, 0, 0, 0, 0)$

(ii) $R_0 > 1 \Rightarrow$ **endemic equilibrium**
P_0 *and* $P^*(S, E, I_P, I_s, I_a, H, R) > \overline{0}$

Variational Iteration Method (VIM)

To clarify the fundamental approach of variational iteration scheme, we examined the differential equation below:

$$Lu + Nu = g(t) \quad (17)$$

Here, L and N represent the linear and nonlinear parameters accordingly, while $g(t)$ denotes the non-uniform source term.

The variational iteration scheme presents a correction functional given by:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda(\varepsilon)\big(Lu_n(\varepsilon) + N\tilde{u}_n(\varepsilon) - g(\varepsilon)\big)d\varepsilon \quad (18)$$

where λ is a universal Lagrange multiplier, that can be obtained by employing the variational principle, and $\tilde{u}_n$ denotes a constrained variation meaning that $\delta\tilde{u}_n = 0$.

We begin with determining the Lagrange multiplier $\lambda(\varepsilon)$ efficiently utilizing product integration. In other words, performing the integration in the following manner yields:

$$\int \lambda(\varepsilon)\, u'_n(\varepsilon)d\varepsilon = \lambda(\varepsilon)u_n(\varepsilon) - \int \lambda'(\varepsilon)\, u_n(\varepsilon)d\varepsilon, \quad (19)$$

Then, the successive numerical solution for u of u_{n+1}, $n \geq 0$, will obtained by applying any appropriate selective function u_0.

However, to obtained rapid convergence, the function $u_0(x,t)$ should be chosen based on the first conditions as follows:

$$u_0(x,t) = u(x,0)$$

for first order

Consequently, the solution

$$u = \lim_{n\to\infty} u_n$$

In other words, equation (19) generates a sequence of approximations, and the exact solution is attained as the limit of these successive approximations [16].

Variational Iteration Method for the model equations

The modification functional for equations (1-7)

$$S_{n+1}(t) = S_n(t) + \int_0^t \lambda_1(\varepsilon)\left[\frac{\partial S_n}{d\varepsilon} - \beta + \gamma\tilde{S}(\tilde{I}_{Pn} + \tilde{I}_{Sn} + \tilde{I}_{an}) + \mu\tilde{S}_n\right]d\varepsilon$$

$$E_{n+1}(t) = E_n(t) + \int_0^t \lambda_2(\varepsilon)\left[\frac{\partial E_n}{d\varepsilon} - \gamma\tilde{S}(\tilde{I}_{Pn} + \tilde{I}_{Sn} + \tilde{I}_{an}) + \widetilde{E_n}(\alpha_1 + \alpha_2 + \alpha_3 + \mu)\right]d\varepsilon$$

$$I_{p(n+1)}(t) = I_{p(n)}(t) + \int_0^t \lambda_3(\varepsilon)\left[\frac{\partial I_{pn}}{d\varepsilon} - \alpha_1\tilde{E}_n - \tilde{I}_{pn}(\alpha_4 + \mu)\right]d\varepsilon$$

$$I_{s(n+1)}(t) = I_{s(n)}(t) + \int_0^t \lambda_4(\varepsilon)\left[\frac{\partial I_{sn}}{d\varepsilon} - \alpha_2\tilde{E}_n - \alpha_4\tilde{I}_{pn} + \tilde{I}_{sn}(\alpha_5 + \mu)\right]d\varepsilon$$

$$I_{a(n+1)}(t) = I_{a(n)}(t) + \int_0^t \lambda_5(\varepsilon)\left[\frac{\partial I_{an}}{d\varepsilon} - \alpha_3\tilde{E}_n + \tilde{I}_{an}(\alpha_6 + \mu)\right]d\varepsilon$$

$$H_{n+1}(t) = H_n(t) + \int_0^t \lambda_6(\varepsilon)\left[\frac{\partial H_n}{d\varepsilon} - \tilde{I}_{sn}\alpha_5 + \widetilde{H}_n(\alpha_7 + \delta + \mu)\right]d\varepsilon$$

$$R_{n+1}(t) = R_n(t) + \int_0^t \lambda_7(\varepsilon)\left[\frac{\partial R_n}{d\varepsilon} - \widetilde{H}_n(\alpha_7 + \delta) - \alpha_6\tilde{I}_{an} + \mu\tilde{R}_n\right]d\varepsilon$$

Applying variations and solving using integrations by parts, we have

$$\lambda_1 = -1, \lambda_2 = -1, \lambda_3 = -1, \lambda_4 = -1, \lambda_5 = -1, \lambda_6 = -1 \; and \; \lambda_7 = -1$$

Therefore the iterative equations become:

$$S_{n+1}(t) = S_n(t) - \int_0^t \left[\frac{\partial S_n}{d\varepsilon} - \beta + \gamma\tilde{S}(\tilde{I}_{Pn} + \tilde{I}_{Sn} + \tilde{I}_{an}) + \mu\tilde{S}_n\right]d\varepsilon \quad (21)$$

$$E_{n+1}(t) = E_n(t)1 - \int_0^t \left[\frac{\partial E_n}{d\varepsilon} - \gamma\tilde{S}(\tilde{I}_{Pn} + \tilde{I}_{Sn} + \tilde{I}_{an}) + \widetilde{E_n}(\alpha_1 + \alpha_2 + \alpha_3 + \mu)\right]d\varepsilon \quad (22)$$

$$I_{p(n+1)}(t) = I_{p(n)}(t) - \int_0^t \left[\frac{\partial I_{pn}}{d\varepsilon} - \alpha_1\tilde{E}_n - \tilde{I}_{pn}(\alpha_4 + \mu)\right]d\varepsilon \quad (23)$$

$$I_{s(n+1)}(t) = I_{s(n)}(t) - \int_0^t \left[\frac{\partial I_{sn}}{d\varepsilon} - \alpha_2 \tilde{E}_n - \alpha_4 \tilde{I}_{pn} + \tilde{I}_{sn}(\alpha_5 + \mu)\right] d\varepsilon \quad (24)$$

$$I_{a(n+1)}(t) = I_{a(n)}(t) - \int_0^t \left[\frac{\partial I_{an}}{d\varepsilon} - \alpha_3 \tilde{E}_n + \tilde{I}_{an}(\alpha_6 + \mu)\right] d\varepsilon \quad (25)$$

$$H_{n+1}(t) = H_n(t) - \int_0^t \left[\frac{\partial H_n}{d\varepsilon} - \tilde{I}_{sn}\alpha_5 + \tilde{H}_n(\alpha_7 + \delta + \mu\right] d\varepsilon \quad (26)$$

$$R_{n+1}(t) = R_n(t) - \int_0^t \left[\frac{\partial R_n}{d\varepsilon} - \tilde{H}_n(\alpha_7 + \delta) - \alpha_6 \tilde{I}_{an} + \mu \tilde{R}_n\right] d\varepsilon \quad (27)$$

Equations (21-27) will yield the estimated solution for the transmission of COVID-19

Numerical Solution of the Model

To solve numerically using the iteration, we need initial data, we will assumed data for the variables and the parameters:

Using assumed variables and parameters:
S(0) = 1000, E(0) = 500, Ip(0) = 50, Is(0) = 20, Ia(0) = 10, H(0) = 10, R(0) = 2
β = 0.5, μ = 0.5, γ = 0.01, δ = 0.02, α_1 = 0.05, α_2 = 0.05, α_3 = 0.05, α_4 = 0.03,
α_5 = 0.03, α_6 = 0.06, α_7 = 0.01
The solutions of the equations after 2nd iterations are:

$$S(t) = 1000 - 1299.5t + 908.18t^2 + 55.01t^3$$
$$E(t) = 500 + 475t - 731.83t^2 + 55.01t^3$$
$$I_p(t) = 50 - 1.5t + 12.27t^2$$
$$I_s(t) = 20 + 15.9t + 7.684t^2$$
$$I_a(t) = 10 + 19.4t + 6.44t^2$$
$$H(t) = 10 - 4.7t + 1.484t^2$$
$$R(t) = 2 - 0.1t + 0.54t^2$$

Graphical Illustrations of the Model and Discussion of Results

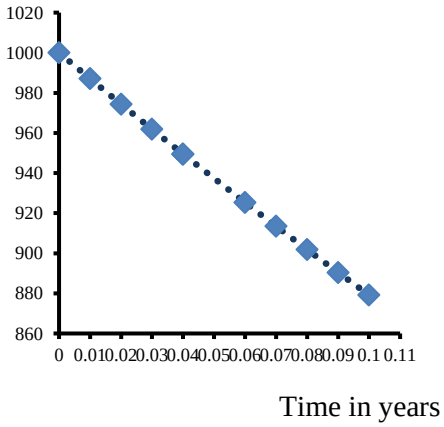


Figure 5: VIM Solution of Susceptible population

Depicted the individuals who are at risk of being infected but have not contacted disease. It is a key component of models like SEIR models and their extension.

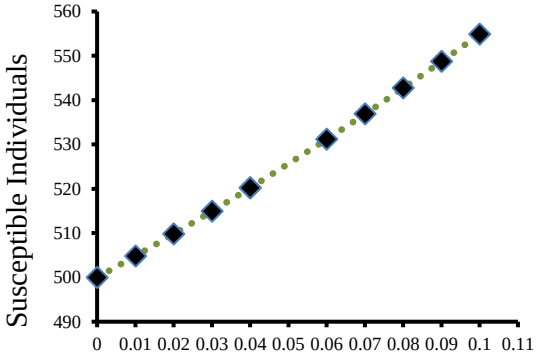


Figure 6: VIM So ulation

The fig. shows the exposed population and it represents individuals who have been infected but are not yet infectious.

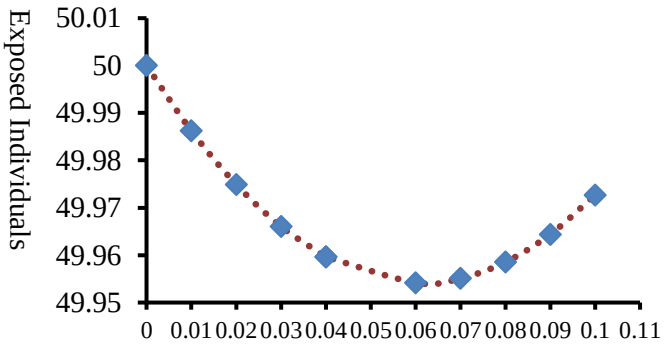
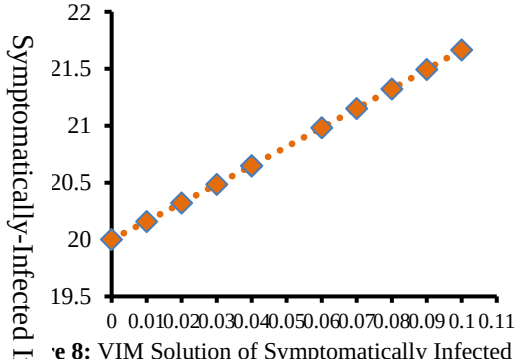


Figure 7: VIM Solution of Pre-symptomatically-Infected population
It depicted the individuals who have been infected but are not yet showing symptoms though they may still be infectious.



e 8: VIM Solution of Symptomatically Infected ation
ws the individuals who have transitioned from the mptomatic or exposed stage. It also shows an ly symptoms and can spread the disease before ery or isolation.

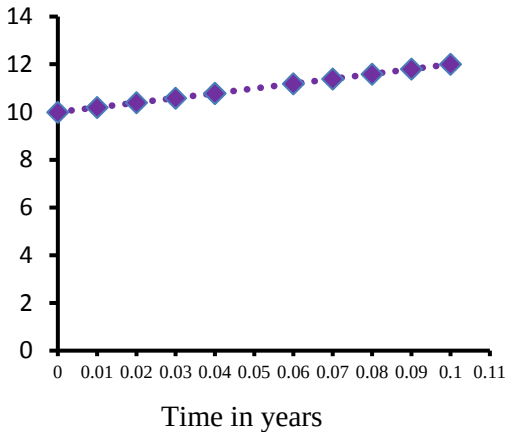


Figure 9: VIM Solution of Asymptomatically Infected population

It shows the susceptible-exposed-infected-asymptotic recovered. It also shows that the individuals have transitioned from the exposed stage but do not develop symptoms but may also spread the disease but may recover at a certain stage.

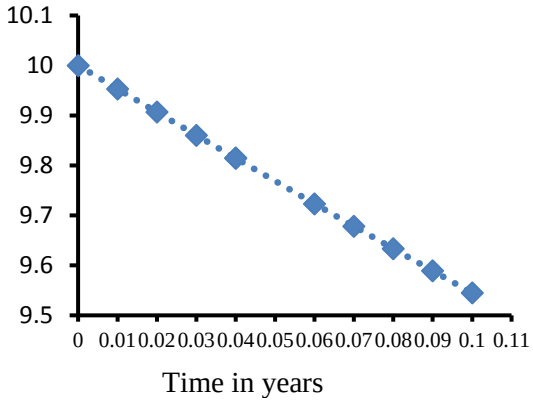


Figure 10: VIM Solution of Hospitalized population

The fig. shows the hospitali t of individuals who have sever hospitalization and also transition from the symptomatic infected class with either recover or die due to the infection.

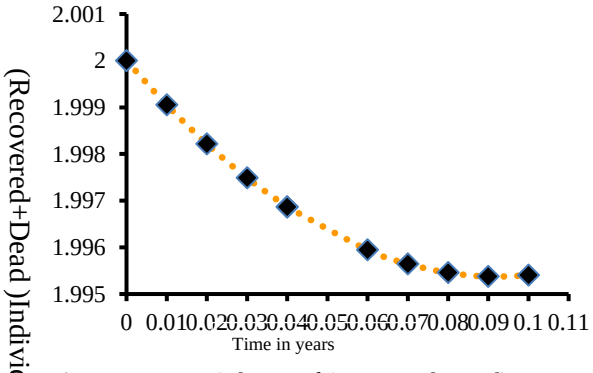


Figure 11: VIM Solution of (Recovered+Dead) population

It shows the dead population who succumbed to the disease and also recovered population who have overcome the infection and gained immunity.

Conclusion

In this study, a modified SEIR model was presented to study the epidemic pattern of COVID-19 Coronavirus. Our model shows the difference between pre-symptomatic and asymptomatic Infectious individual. We have been able to determine the feasible region for the positive solution of the pandemic using our model, the equilibrium points in the feasible region and to derive the general reproductive number for the system i.e. if $R_0 \leq 1$, then we have a no effect equilibrium but if $R_0 > 1$, then an endemic equilibrium is established.

Variational iteration method (VIM) has been employed to give the approximate solution of the proposed model. Our results demonstrate that variational iteration method is reliable and effective approach for analysing both linear and nonlinear problems.

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